

Optimization and Modal Wave Propagation Analysis of Dielectric Tube Waveguide Loaded with Plasma- TLBO

R R Hirani*[#], S N Shah[#], S K Pathak[†]

*HJD Institute, Kutch, India, hirani.rasila@gmail.com, [#]SVNIT, Surat, India, [†] Institute for Plasma Research, India

Abstract

The Dielectric Tube Waveguide Loaded with Plasma (DTWLP) is an open waveguide structure that shows surface wave propagation characteristics. In this paper, an analytical theory of dispersion for DTWLP has been investigated numerically for higher order Hybrid modes (HE and EH) for guided mode. The appropriate value of dielectric constant, Plasma wave number and core to cladding ratio were found out using Teaching Learning Based Optimization (TLBO) with an aim to get wide single mode wave propagation bandwidth. This type of waveguide is loaded with plasma get an advantage of plasma reconfigurable property, where retuning of waveguide is required. Also, DTWLP can be used as a guided mode antenna where radiation is coming out from the end of waveguide. It can be used as a highly directive end fire antenna and potential application in stealth technology.

Keywords: Dispersion, Hybrid Mode, Optimization, Plasma, TLBO.

1 Introduction

In the recent years, plasma loaded cylindrical waveguide has been growing interest in cylindrical waveguides [1–4]. Many researchers have done the study on losses of waveguide based on material and geometrical parameter. With the development of high-frequency electronics, interest in analyzing, studying and understanding plasma-filled waveguides has been growing with the intention of creating powerful microwave generators. Cylindrical waveguides are employed in microwave sources, consisting of axis encircling electron beams. This subject is currently of significant interest in the development of optical systems, high information density communication, RF foil of particle accelerators, fusion experiments and high-power millimeter wave amplifiers. Also, DTWLP supports open boundary condition therefore it can be used as a surface wave antenna. When DTWLP is excited with a pulsed RF power, the plasma is ionized and behave as a good conductor, therefore it would be transmitting signal like an antenna, when it is off, there is only a minor reflection from the dielectric tube, and hence it has a very low RCS used in stealth technology [5].

In the waveguide, DTWLP has three regions as shown in Fig.1. Inner region is a frequency dependent plasma material having a radius (r_1) and permittivity ϵ_1 . Plasma is covered with dielectric material having a radius (r_2) and permittivity ϵ_2 . Outer most region ($r \rightarrow \infty$) is a free space region having permittivity ϵ_3 . DTWLP has permittivity profile ($\epsilon_2 > \epsilon_3 > \epsilon_1$). Such waveguide structures have a non-zero cut off value for HE mode and has potential to provide larger bandwidth in single mode propagation [6].

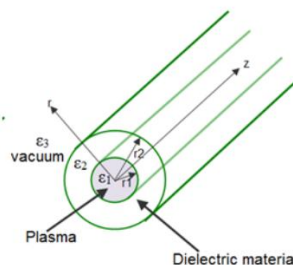


Fig. 1 Schematic Diagram of Dielectric Tube Loaded with Plasma (DTWLP)

The objective of this study is to identify the different modes of propagation and their dispersion relation (ω - β). The wave vector, β gives the direction of propagation of the wave. An EM wave propagating at $\omega < \omega_p$ is able to excite plasma oscillations within the plasma. However, when $\omega > \omega_p$, EM waves propagate through plasma in guided mode.

In this study, our interest is to analyze the plasmaloading effect on propagation characteristics and the dispersion relation. In lossless plasma, when $\omega > \omega_p$ the propagation constant should be purely real, allowing the EM wave to freely propagate without any attenuation. When the plasma medium is bounded by a dielectric material ϵ_2 , a surface wave is propagating along with evanescent fields on both sides of the boundary. The plasma permittivity can be defined as:

$$\epsilon_p = 1 - \frac{\omega_p^2}{\omega_0^2} \tag{1}$$

$$\epsilon_p = 1 - \frac{\omega_p^2 r_1^2 / c^2}{\omega_0^2 r_1^2 / c^2} = 1 - \frac{(K_p r_1)^2}{(K_0 r_1)^2}$$

where, r_1 is the radius of the inner core plasma column, $K_p = \omega_p / c$ is the plasma wave number, $K_0 = \omega_0 / c$ is the free space wave number, c is speed of light and ω_p is plasma angular frequency defined as,

$$\omega_p = \sqrt{\frac{Ne^2}{m\epsilon_0}} \tag{2}$$

where N is plasma density, ϵ_0 is the permittivity of the free space and e and m are charge and mass of an electron respectively.

2 Mathematical Formulation of DTWLP and TLBO

2.1 Characteristics Equation

The mathematical theory of dielectric tube waveguide is comprehensively derived by Safaai-Jazi et al. [7] using different Bessel function situations for modal characteristics of electromagnetic wave propagation. The characteristic equation for DTWLP was derived in detail by our previous research paper [3]. Consider a cylinder co-ordinate system (r, θ, z) where z is axial wave propagation direction. The axial components of electric and magnetic field in each region according to [8] can be expressed as

$$E_{z1} = A_1 I_m(K_1 r) e^{j(\omega t - \beta z - m\theta)} \tag{3}$$

$$H_{z1} = B_1 I_m(K_1 r) e^{j(\omega t - \beta z - m\theta)} \tag{4}$$

$$E_{z2} = [A_2 J_m(K_2 r) + A_3 Y_m(K_2 r)] e^{j(\omega t - \beta z - m\theta)} \tag{5}$$

$$H_{z2} = [B_2 J_m(K_2 r) + B_3 Y_m(K_2 r)] e^{j(\omega t - \beta z - m\theta)} \tag{6}$$

$$E_{z3} = A_4 K_m(K_3 r) e^{j(\omega t - \beta z - m\theta)} \tag{7}$$

$$H_{z3} = B_4 K_m(K_3 r) e^{j(\omega t - \beta z - m\theta)} \tag{8}$$

J_m and Y_m are the Bessel function of first kind and second kind, I_m and K_m are the modified Bessel functions of first and second kind respectively.

The wave propagation constant in transverse direction for all the mediums are given as [8]

$$K_1^2 = K_0^2 (\bar{\beta}^2 - \mu_1 \epsilon_1) \tag{9}$$

$$K_2^2 = K_0^2 (\mu_2 \epsilon_2 - \bar{\beta}^2) \tag{10}$$

$$K_3^2 = K_0^2 (\bar{\beta}^2 - \mu_3 \epsilon_3). \tag{11}$$

The radial and azimuthally electric and magnetic field components can be found in terms of the z components by Maxwell's curl equations [9]. By applying the boundary condition and derived the dispersion relation as mentioned in [3].

$$G_1 \eta_1^2 + G_2 \eta_1 + G_3 = 0 \tag{12}$$

2.2 Teaching Learning Based Optimization (TLBO)

TLBO is an optimization method developed by Rao et al. [9]. It is an algorithm similar to other nature inspired evolutionary algorithm which works on the typical classroom teaching method. TLBO comprises of two phases: (1) Teacher Phase (2) Learner Phase. In the teacher phase, learners first acquire information from a teacher in order to

increase the mean result of the class. In this phase, learners learn from the teacher and the teacher tries to enhance the result of the other individual (P_i) by increasing the mean result of the classroom (P_{mean}) towards the result of the teacher ($P_{teacher}$). Two random variables are generated in the range 0 and 1, which is stored in the variable, r . [10]:

$$P_{new} = P_i + r.(P_{teacher} - T_f.P_{mean}) \tag{13}$$

where P_{new} and P_i are the new and existing solution of an i^{th} individual, and T_f is a teaching factor which can either be 1 or 2.

In the learner phase, algorithm simulates the learning of the students through mutual interaction, where students gain information by discussing and interacting with each other. A learner will obtain new information from other learners possessing more knowledge than him. During this stage, the student P_i interacts with another student P_j randomly in order to enhance his/her knowledge. After learning, if P_j possesses more knowledge than P_i then P_i is moved towards P_j .

$$P_{new} = P_i + r (P_i - P_j) \text{ if } f(P_i) < f(P_j) \tag{14}$$

$$P_{new} = P_i + r (P_j - P_i) \text{ if } f(P_j) < f(P_i) \tag{15}$$

The improved new solution, P_{new} is accepted in the population and this algorithm will continue until the best-fit condition is obtained. Due to its ease in implementation and high efficiency, TLBO has become a very popular optimization algorithm and has been productively exploited to numerous real world problems. This paper implements the algorithm for 6 students.

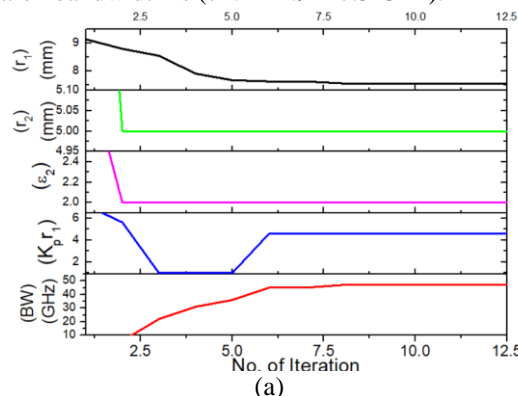
3 Simulation Results and Discussions

To obtain the normalized propagation constant (β/K_0) as a function of plasma frequency, the dispersion relation is solved numerically by complex root search Muller method using MATLAB. The dispersion equation (12) is used here as fitness function of TLBO. The dispersion characteristics for the lower order of hybrid mode (HE and EH) are plotted in Fig. 2 and 3. TLBO algorithm is used to optimize the guided mode characteristics for DTWLP. Here the population (No. of Students) is 6 and design parameters are radius of the core (r_1) and cladding (r_2), plasma wave number (K_{pr1}) and the dielectric constant of the tube (ϵ_2). These four design parameters are optimized in TLBO algorithm to find out the best possible output for different operating modes (such as HE and EH). The main objective is to find out wide single mode propagation bandwidth. Now take the following design parameter to optimize the single mode propagation bandwidth for HE and EH mode.

- Core Radius (r_1) = 5 mm to 8 mm
- Cladding Radius (r_2) = 7.5 to 12.5 mm
- Plasma Wave Number (K_{pr1}) = 1 to 8
- Dielectric Constant (ϵ_2) = 2 to 8

Within 20 iterations TLBO gives the final/ mature solution, this is the beauty of this optimization algorithm it is fast converged algorithm.

Fig. 2 (a) shows the optimized data of each design variables with each solution for HE_{1n} mode. This shows that dielectric constant, core and cladding radius is converged at the lower limit of optimization design data. The final solution is large single mode propagation bandwidth. Fig. 2 (b) shows the dispersion characteristics of the HE_{1n} mode with optimized parameter are $r_1 = 5$ mm, $r_2 = 7.5$ mm, $\epsilon_2 = 2$ and $K_{pr1} = 4.98$. The cutoff frequency for HE_{11} is 24.9 GHz and HE_{12} is 72.4 GHz. The single mode propagation bandwidth is (72.4-24.9=47.5 GHz).



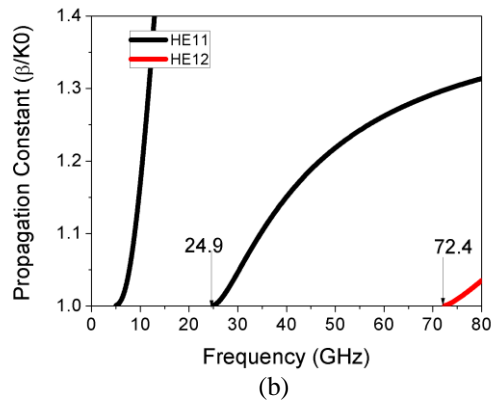


Fig. 2: (a) TLBO optimization output for HE mode: Design variable Vs. no. of iteration (b) Dispersion characteristics of HE_{1n} mode with optimized design variable are $r_1 = 5$ mm, $r_2 = 7.5$ mm, $\epsilon_2 = 2$, $K_p r_1 = 4.56$

In Fig. 3 (a), it is shown the optimized data for EH_{1n} Mode. Fig. 3 (b) shows the dispersion characteristics of EH_{1n} mode with optimized parameter are $r_1 = 5$ mm, $r_2 = 7.5$ mm, $\epsilon_2 = 2$ and $K_p r_1 = 1$. The cutoff frequency for EH_{11} is 30.9 GHz and EH_{12} is 75.3 GHz. The single mode propagation bandwidth is 44.4 GHz.

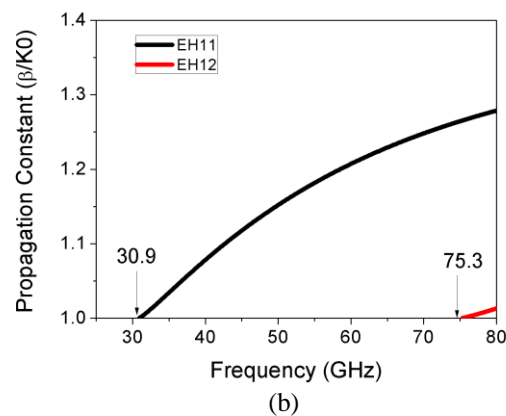
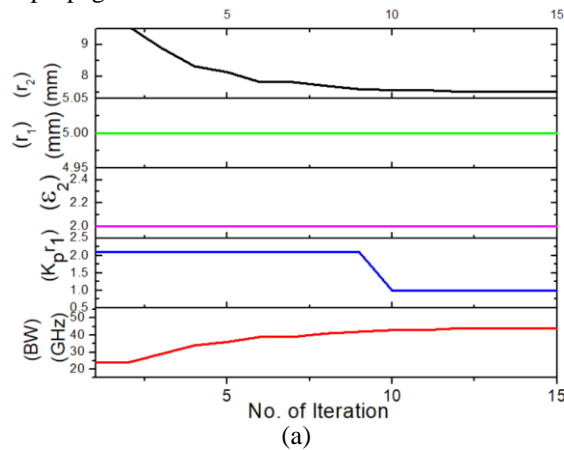
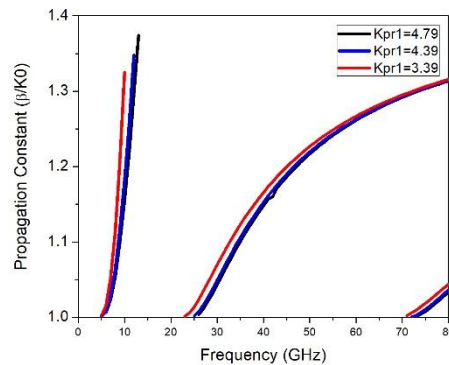


Fig. 3: (a) TLBO optimization output for EH mode: Design variable Vs. no. of iteration (b) Dispersion characteristics of EH_{1n} mode with optimized design variable are $r_1 = 5$ mm, $r_2 = 7.5$ mm, $\epsilon_2 = 2$, $K_p r_1 = 1$

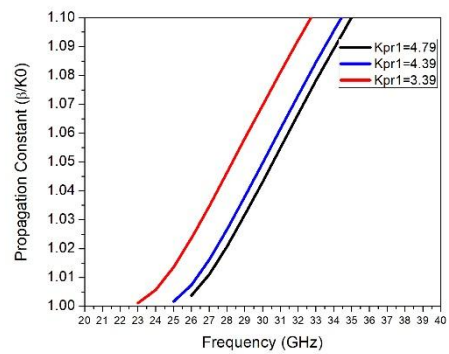
A. Effect of Plasma Wave Number ($K_p r_1$)

The main advantage of plasma waveguide is that; it has reconfigurable property. By the electrically change in the plasma density it will change the plasma frequency dependent permittivity. Over all permittivity it will change the cutoff value of modal equation. Therefore, the waveguide is tuned at desired cutoff value. Here for HE mode, TLBO is applied to dispersion equation. At each time it will converge at random value of plasma wavenumber with the highest bandwidth. So, TLBO computation was done for 3 times and generated three different solutions each time bandwidth and core to

cladding radius were same but plasma wave number was different. With the plotting of optimized data in Fig. 4 shows, dispersion characteristics of HE_{1n} mode with the variation in cutoff value. Fig. 4 (b) shows the enlarged version of 4 (a). It is clearly shown the cut off value of HE₁₁ mode is different for plasma wave number.



(a)



(b)

Fig. 4: (a) Dispersion characteristics for HE_{1n} with a change in plasma wave number ($K_p r_1$). Here ϵ_2 is 2 and r_2/r_1 is 1.5. (b) enlarged view of fig. 4 (a) to show the variation in the cutoff value of HE₁₁

HE ₁₁ (GHz)	HE ₁₂ (GHz)	Core to Cladding Ratio (r_2/r_1)	Plasma Wave Number ($K_p r_1$)	BW (GHz)
26	73	1.5	4.79	47
25	72	1.5	4.39	47
23	71	1.5	3.39	48

Table 1: Cutoff value for HE mode of DTWLP with variation in plasma wave number ($K_p r_1$)

Table 1 shows the cutoff value of each iteration and bandwidth. Higher the value of plasma wavenumber it will lead the cutoff value of HE mode high. Based on our application we can optimize the value of plasma wavenumber and tune the waveguide at the desired frequency band.

4 Conclusion

The observation taken from DTWLP is that TLBO optimization was converged at a lower limit of the radius and dielectric constant value. Multi-mode propagation losses are reduced and energy is concentrated in single mode for wide frequency range. By filling plasma in the core region, it makes waveguide reconfigurable by electrically tuned plasma density. Modification in plasma density varies the plasma frequency as well as the effective permittivity change which in turn changes the wave propagation behaviour of the waveguide. Normalized plasma frequency should be in between 3 to 4.5 for wide mono mode propagation. It has single mode propagation bandwidth for HE mode is 47.5 GHz and for EH mode is 44.4 GHz. This type of waveguide is used in the high-frequency gyrotron and amplifier, also can be used as an

antenna in RADAR system where it is hidden from hostile antenna by retuning at desired frequency. This perspective is our future research work.

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